極限.

(limit.)

$$\lim_{x \to 2} x = 2$$

(當 x 趨近 2 時, x 會趨近多少.)

$x \to 2$ $(x \neq 2)$

例:

1. $\lim_{x \to 2} x - 2 = 2 - 2 = 0$

2. $\lim_{x \to 0} x - 2 = 0 - 2 = -2$

3. $\lim_{x \to 2} 2x = 2 \times 2 = 4$.

} x 會被數字代換.

4. $\lim_{x \to 2} xy = 2y$

5. $\left(\lim_{x \to 2} x\right) + \left(\lim_{x \to 3} x\right) = 5$

2 3

(A) 4 (B) 5 (C) 6.

- $\lim_{x\to 2}$ 看 x 趋近 2 时的极限

$(x\to 2)$ ✱✱ $x\neq 2$

例: $\lim_{\substack{x\to 2\\ x\neq 2}} \frac{2(x-2)}{(x-2)} = \lim_{x\to 2} 2 = 2$ ← 因为 $x\neq 2$, $x-2\neq 0$, 可约分.

(想: x 用 2 代入? $\frac{2\times 0}{0} = \frac{0}{0}$ ✗)

x 不能用 2 代入!!

c.f. $\frac{2(2-2)}{2-2}$ 不能算

$(a^2-b^2)=(a+b)(a-b)$

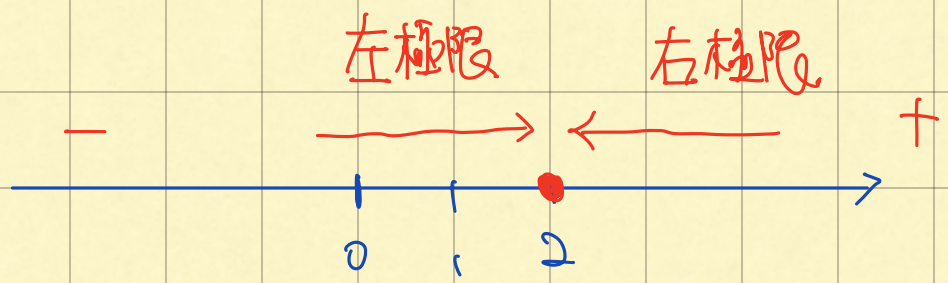
练: 1. $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{\substack{x\to 2\\ (x\neq 2)}} \frac{(x+2)(x-2)}{x-2} = \lim_{x\to 2}(x+2) = 4$

2. $\lim_{x\to 0} \frac{x^{10000}}{x} = \lim_{x\to 0} x^{9999} = 0$

3. $\lim_{x\to 5} \frac{(x-5)(x^2+x+1)}{x-5} = \lim_{x\to 5} x^2+x+1$

$= 5^2+5+1 = 31$

左极限 & 右极限 $(x \to 2)$



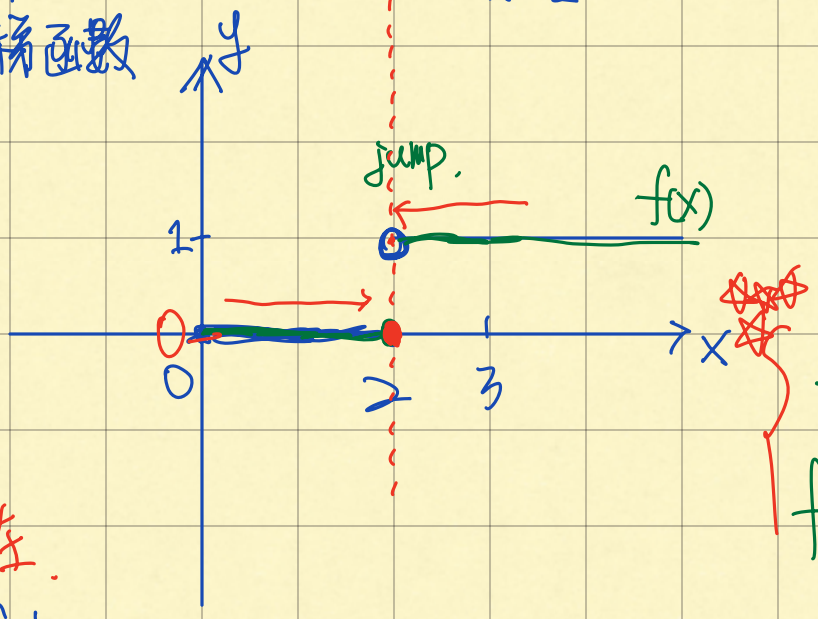
$(x \to 2^-)$ x从左边趋近2

$(x \to 2^+)$ x从右边趋近2

例: $\lim\limits_{x \to 2} x \Rightarrow \begin{cases} \lim\limits_{x \to 2^-} x = 2 & \text{左极限} \\ \lim\limits_{x \to 2^+} x = 2 & \text{右极限} \end{cases}$

step function.
阶梯函数

例



$f(0) = 0$

$f(1) = 0$

$f(2) = 0$

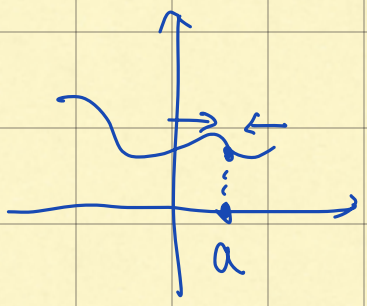
$f(2.0000001) = 1$

? 极限不存在.

$\lim\limits_{x \to 2} f(x) = ?$

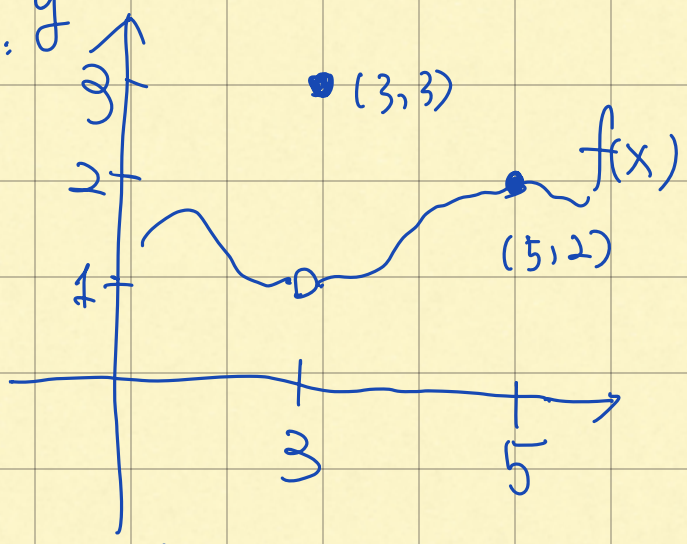
$\begin{cases} \lim\limits_{x \to 2^-} f(x) = 0 \approx f(1.9999999) \quad f(3) = 1 & (x<2) \\ \lim\limits_{x \to 2^+} f(x) = 1 \approx f(2.0000001) & (x>2) \end{cases}$

1° 若 $\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x) = L$，则 $\lim\limits_{x\to a} f(x) = L$



2° 若 $\lim\limits_{x\to a^-} f(x) \neq \lim\limits_{x\to a^+} f(x)$，则 $\lim\limits_{x\to a} f(x)$ 不存在.

例：



1° $f(3) =$

$f(5) =$

2° $\lim\limits_{x\to 3^-} f(x)$

$\lim\limits_{x\to 3^+} f(x)$

$\lim\limits_{x\to 5^-} f(x)$

$\lim\limits_{x\to 5^+} f(x)$

3° $\lim\limits_{x\to 3} f(x)$ 存在吗？

$\lim\limits_{x\to 5} f(x)$ 存在吗？

4° $\lim\limits_{x\to 3} f(x) = f(3)$？

$\lim\limits_{x\to 5} f(x) = f(5)$？